

General Certificate of Education  
June 2007  
Advanced Subsidiary Examination



**MATHEMATICS**  
**Unit Further Pure 1**

**MFP1**

Wednesday 20 June 2007 1.30 pm to 3.00 pm

**For this paper you must have:**

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Questions 5 and 9 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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1 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & 8 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

The matrix  $\mathbf{M} = \mathbf{A} - 2\mathbf{B}$ .

(a) Show that  $\mathbf{M} = n \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ , where  $n$  is a positive integer. (2 marks)

(b) The matrix  $\mathbf{M}$  represents a combination of an enlargement of scale factor  $p$  and a reflection in a line  $L$ . State the value of  $p$  and write down the equation of  $L$ . (2 marks)

(c) Show that

$$\mathbf{M}^2 = q\mathbf{I}$$

where  $q$  is an integer and  $\mathbf{I}$  is the  $2 \times 2$  identity matrix. (2 marks)

2 (a) Show that the equation

$$x^3 + x - 7 = 0$$

has a root between 1.6 and 1.8. (3 marks)

(b) Use interval bisection **twice**, starting with the interval in part (a), to give this root to one decimal place. (4 marks)

3 It is given that  $z = x + iy$ , where  $x$  and  $y$  are real numbers.

(a) Find, in terms of  $x$  and  $y$ , the real and imaginary parts of

$$z - 3iz^*$$

where  $z^*$  is the complex conjugate of  $z$ . (3 marks)

(b) Find the complex number  $z$  such that

$$z - 3iz^* = 16 \quad \text{(3 marks)}$$

## 4 The quadratic equation

$$2x^2 - x + 4 = 0$$

has roots  $\alpha$  and  $\beta$ .

(a) Write down the values of  $\alpha + \beta$  and  $\alpha\beta$ . (2 marks)

(b) Show that  $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{1}{4}$ . (2 marks)

(c) Find a quadratic equation with integer coefficients such that the roots of the equation are

$$\frac{4}{\alpha} \text{ and } \frac{4}{\beta} \quad (3 \text{ marks})$$

## 5 [Figure 1 and Figure 2, printed on the insert, are provided for use in this question.]

The variables  $x$  and  $y$  are known to be related by an equation of the form

$$y = ab^x$$

where  $a$  and  $b$  are constants.

The following approximate values of  $x$  and  $y$  have been found.

$x$	1	2	3	4
$y$	3.84	6.14	9.82	15.7

(a) Complete the table in **Figure 1**, showing values of  $x$  and  $Y$ , where  $Y = \log_{10} y$ .  
Give each value of  $Y$  to three decimal places. (2 marks)

(b) Show that, if  $y = ab^x$ , then  $x$  and  $Y$  must satisfy an equation of the form

$$Y = mx + c \quad (3 \text{ marks})$$

(c) Draw on **Figure 2** a linear graph relating  $x$  and  $Y$ . (2 marks)

(d) Hence find estimates for the values of  $a$  and  $b$ . (4 marks)

**Turn over** ►

6 Find the general solution of the equation

$$\sin\left(2x - \frac{\pi}{2}\right) = \frac{\sqrt{3}}{2}$$

giving your answer in terms of  $\pi$ .

(6 marks)

7 A curve has equation

$$y = \frac{3x - 1}{x + 2}$$

(a) Write down the equations of the two asymptotes to the curve.

(2 marks)

(b) Sketch the curve, indicating the coordinates of the points where the curve intersects the coordinate axes.

(5 marks)

(c) Hence, or otherwise, solve the inequality

$$0 < \frac{3x - 1}{x + 2} < 3$$

(2 marks)

8 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a)  $\int_0^1 (x^{\frac{1}{3}} + x^{-\frac{1}{3}}) dx;$

(4 marks)

(b)  $\int_0^1 \frac{x^{\frac{1}{3}} + x^{-\frac{1}{3}}}{x} dx.$

(4 marks)

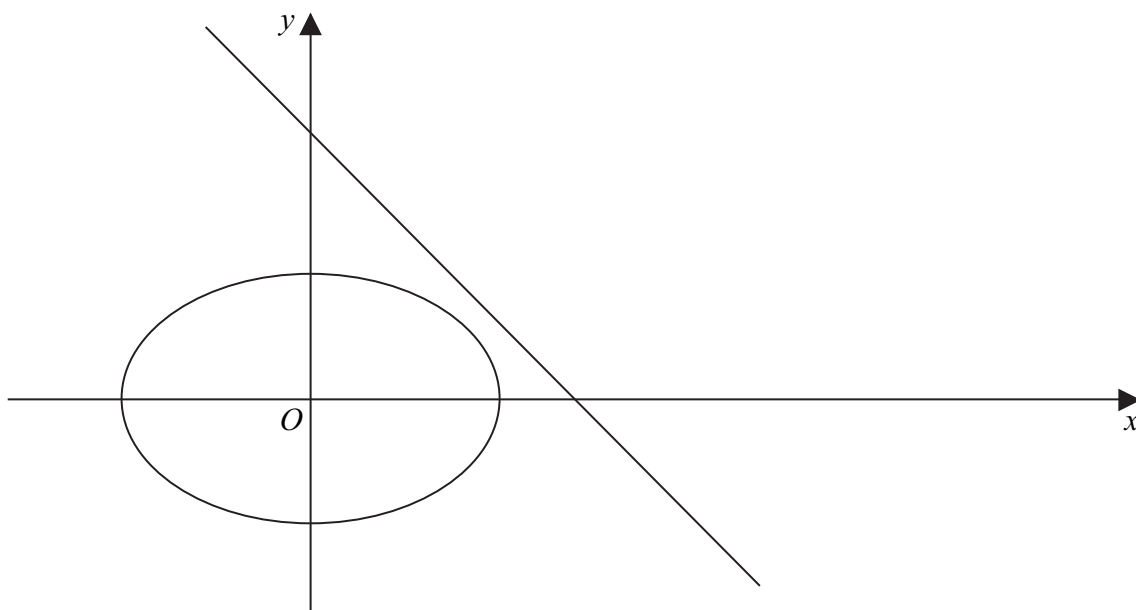
9 [Figure 3, printed on the insert, is provided for use in this question.]

The diagram shows the curve with equation

$$\frac{x^2}{2} + y^2 = 1$$

and the straight line with equation

$$x + y = 2$$



- (a) Write down the exact coordinates of the points where the curve with equation  $\frac{x^2}{2} + y^2 = 1$  intersects the coordinate axes. (2 marks)
- (b) The curve is translated  $k$  units in the positive  $x$  direction, where  $k$  is a constant. Write down, in terms of  $k$ , the equation of the curve after this translation. (2 marks)
- (c) Show that, if the line  $x + y = 2$  intersects the **translated** curve, the  $x$ -coordinates of the points of intersection must satisfy the equation
- $$3x^2 - 2(k + 4)x + (k^2 + 6) = 0 \quad (4 \text{ marks})$$
- (d) Hence find the two values of  $k$  for which the line  $x + y = 2$  is a tangent to the translated curve. Give your answer in the form  $p \pm \sqrt{q}$ , where  $p$  and  $q$  are integers. (4 marks)
- (e) On **Figure 3**, show the translated curves corresponding to these two values of  $k$ . (3 marks)

**END OF QUESTIONS**

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Surname						Other Names					
Centre Number						Candidate Number					
Candidate Signature											

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**MFP1**

# Insert

Insert for use in **Questions 5 and 9**.

Fill in the boxes at the top of this page.

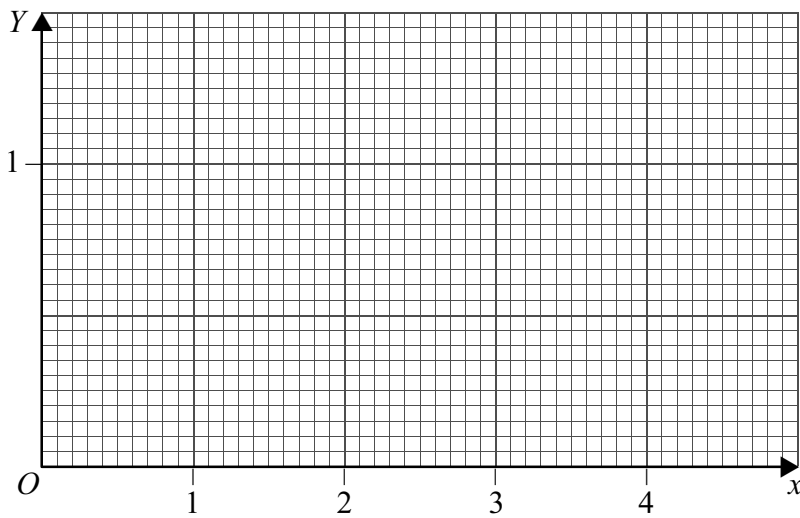
Fasten this insert securely to your answer book.

**Turn over for Figure 1**

**Turn over ►**

**Figure 1 (for use in Question 5)**

$x$	1	2	3	4
$Y$	0.584			

**Figure 2 (for use in Question 5)****Figure 3 (for use in Question 9)**